
High-Level vs. RTL Combinational Equivalence: An Introduction

Alan J. Hu

University of British Columbia

Outline

- Motivation, Problem Statement
- Gate-Level Equivalence Verification
 - Symbolic Simulation
 - Cutpoints
- Symbolic Simulation of a High-Level Model
- Early Cutpoint Insertion
- Future Directions

Why verify?

- Bugs are expensive.
- Bugs are so expensive that:
 - ❑ Verification is primary front-end productivity bottleneck.
 - ❑ Verification costs swamp design costs.

Why formally verify?

- Simulation speed growing exponentially, with Moore's Law.
- Design size also growing exponentially.
- Therefore, possible behaviors growing **doubly** exponentially!

- Behavior coverage from simulation and testing have become unacceptably low.

What's formal verification?

- Formal verification means **proving** a **property** about a **model** of a design.
 - “**proving**” – as good as mathematical proof.
 - “**property**” – got to specify what is correct
 - “**model**” – the tool runs at some level (layout, schematic, RTL, etc.)
- In the past 15-20 years, revolutionary new ideas make formal verification practical in many cases

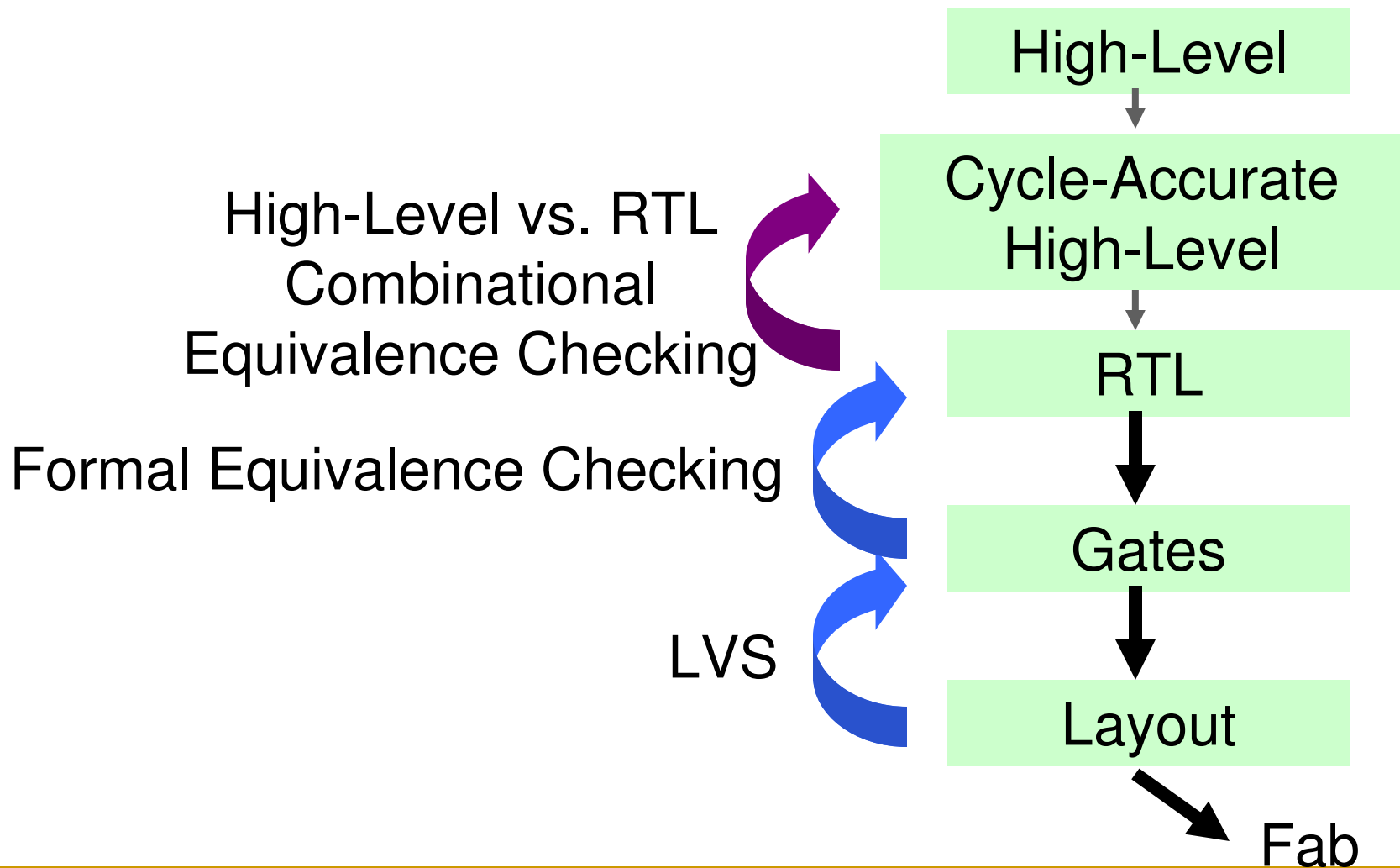
Two Kinds of Verification

- Property Checking (aka Design Verification)
 - Formally specify desired properties (e.g., mutual exclusion, no deadlock)
 - Check that model satisfies property (model checking)
- Equivalence Checking (aka Implementation Verification)
 - Check whether two models are equivalent
 - Biggest success of formal verification to date

Why equivalence checking?

- Theoretically, two kinds of verification are the same.
- In practice, they are different:
 - No separate specification needed
 - Assumption of similarity between two designs

Why high-level vs. RTL?



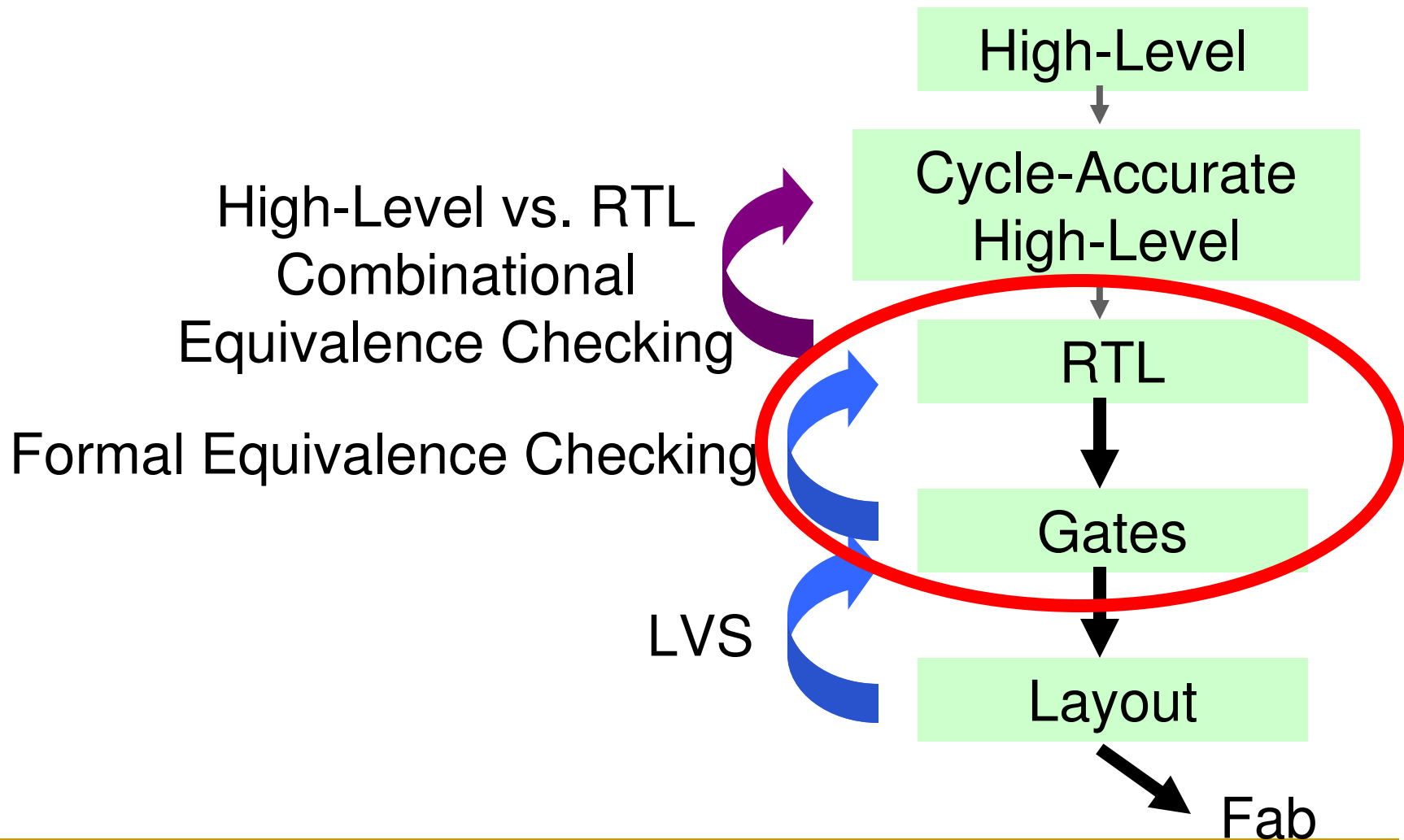
Problem Statement

Given

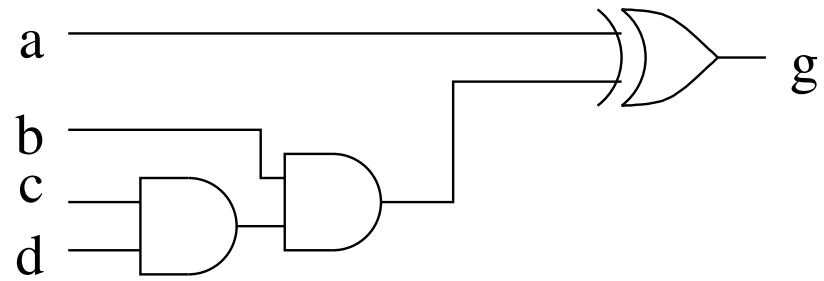
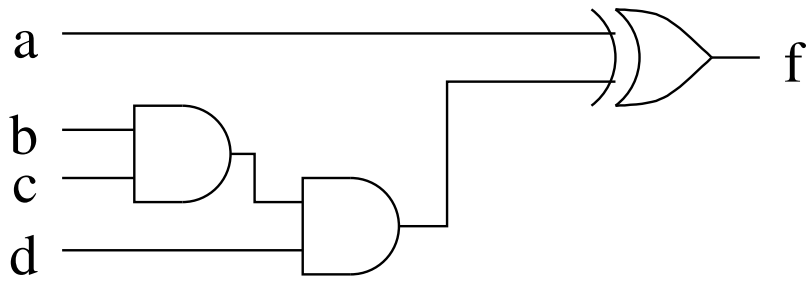
- A high-level software model
 - “Combinational” – output as function of inputs
 - “Non-synthesizable” – too complex for current tools
- A combinational hardware model

Do they have the same functionality?

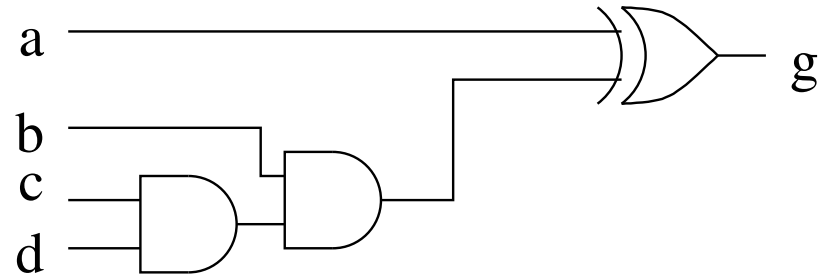
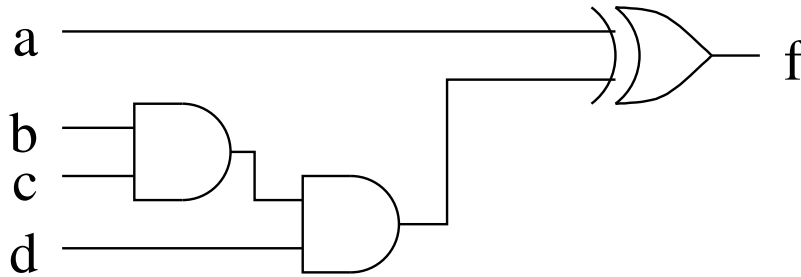
Equivalence Checking



Combinational Equivalence



Combinational Equivalence



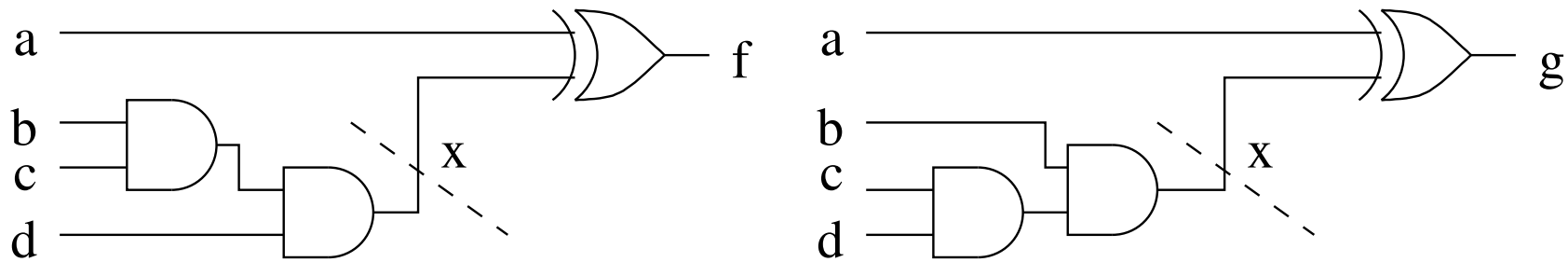
Symbolically simulate both circuits

$$f = a \oplus ((b \wedge c) \wedge d) \quad g = a \oplus (b \wedge (c \wedge d))$$

Compare results (BDDs, SAT, etc.)

Complexity blow up for industrial circuits.

Cutpoints



Guess cutpoint and prove equivalence:

E.g., the wire x in each circuit

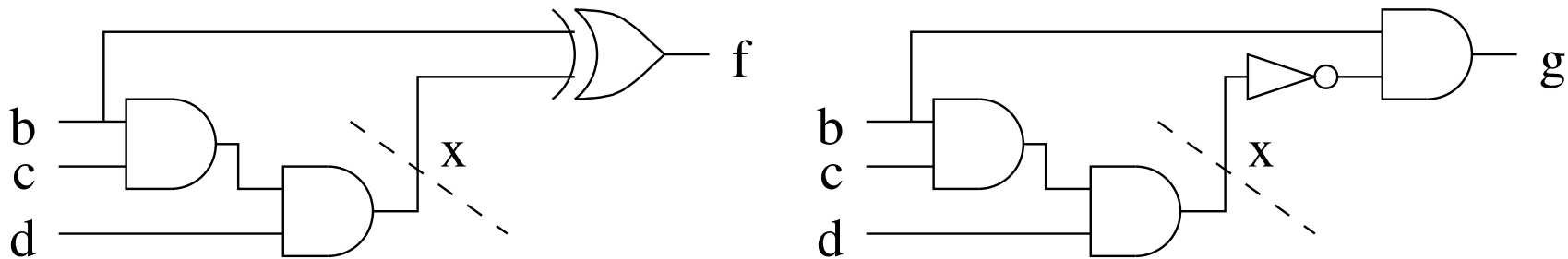
Prove $((b \wedge c) \wedge d) = (b \wedge (c \wedge d))$

Treat cutpoint as new primary input:

Prove $f = a \oplus x$ equivalent to $g = a \oplus x$

Divide and conquer.

False Inequivalence



Guess cutpoint and prove equivalence:

E.g., the wire x in each circuit

Treat cutpoint as new primary input:

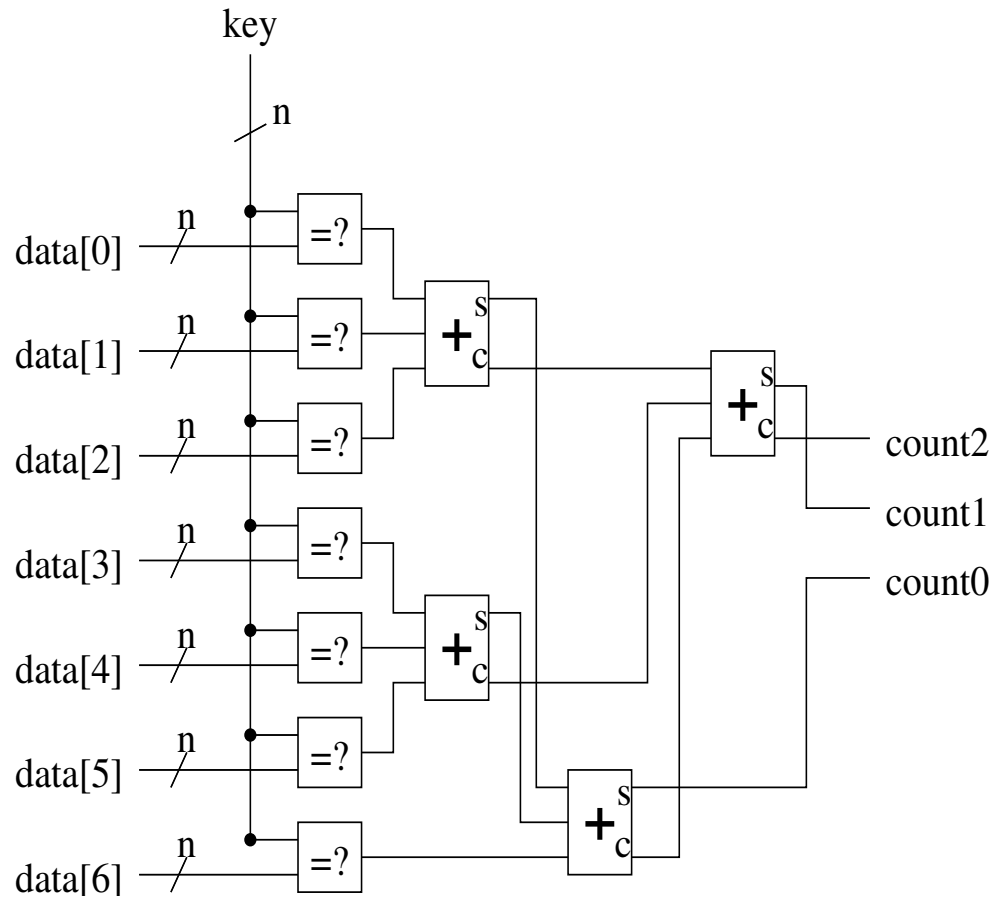
$f = b \oplus x$ versus $g = b \wedge (!x)$

Combinational Equivalence: Key Ideas

- Symbolically simulate to compute functionality.
- Use an efficient representation for the symbolic simulation, e.g., BDDs or circuit-like structure for SAT.
- Find equivalent points to use as cutpoints to simplify the problem.

Software vs. Hardware

```
int f(int key, int data[7])  
{  
    int i, count = 0;  
    for (i=0; i<7; i++) {  
        if (key==data[i])  
            count++;  
    }  
    return count;  
}
```



Combinational Equivalence: Key Ideas

- **Symbolically simulate to compute functionality.**
- Use an efficient representation for the symbolic simulation, e.g., BDDs or circuit-like structure for SAT.
- Find equivalent points to use as cutpoints to simplify the problem.

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data
- i = ?

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data
- i = ?
- count = 0

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data
- i = 0
- count = 0

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data
- i = 0
- count = 0

Assume:

orig_key==orig_data[0]

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data
- i = 0
- count = 1

Assume:

orig_key==orig_data[0]

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- key = orig_key
- data = orig_data
- i = 1
- count = 2

Assume:

```
orig_key==orig_data[0]
orig_key==orig_data[1]
```

Software vs. Hardware

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- Different results on every path
- Must track assumptions on each path
- Exponential number of paths!
- Merge paths with conditional expressions?

Path Merging

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- $i = 0$
- $\text{count} = (\text{orig_key} == \text{orig_data}[0]) ? 1 : 0$

Path Merging

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- $i = 1$
- $\text{count} =$
 $(\text{orig_key} == \text{orig_data}[1])$
 $?$
 $(\text{orig_key} == \text{orig_data}[0])$
 $?2:1) :$
 $(\text{orig_key} == \text{orig_data}[0])$
 $?1:0)$

Path Merging

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- $i = 2$
- $\text{count} =$
 $((\text{orig_key} == \text{orig_data}[2]) ? ($
 $(\text{orig_key} == \text{orig_data}[1]) ?$
 $((\text{orig_key} == \text{orig_data}[0]) ? 3 : 2$
 $) :$
 $((\text{orig_key} == \text{orig_data}[0]) ? 2 : 1$
 $)) : ((\text{orig_key} == \text{orig_data}[1])$
 $?$
 $((\text{orig_key} == \text{orig_data}[0]) ? 2 : 1$
 $) :$
 $((\text{orig_key} == \text{orig_data}[0]) ? 1 : 0$
 $))$

Path Merging

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- $i = 3$
- $\text{count} = (\text{orig_key} == \text{orig_data}[3])$
 $? (\text{orig_key} == \text{orig_data}[2]) ?$
 $(\text{orig_key} == \text{orig_data}[1]) ?$
 $((\text{orig_key} == \text{orig_data}[0]) ? 3 : 2) :$
 $((\text{orig_key} == \text{orig_data}[0]) ? 2 : 1)) :$
 $(\text{orig_key} == \text{orig_data}[1]) ?$
 $((\text{orig_key} == \text{orig_data}[0]) ? 2 : 1) :$
 $((\text{orig_key} == \text{orig_data}[0]) ? 1 : 0)))$
 $: (\text{orig_key} == \text{orig_data}[2]) ?$
 $(\text{orig_key} == \text{orig_data}[1]) ?$
 $((\text{orig_key} == \text{orig_data}[0]) ? 3 : 2) :$
 $((\text{orig_key} == \text{orig_data}[0]) ? 2 : 1)) :$
 $(\text{orig_key} == \text{orig_data}[1]) ?$
 $((\text{orig_key} == \text{orig_data}[0]) ? 2 : 1) :$
 $((\text{orig_key} == \text{orig_data}[0]) ? 1 : 0)))$

Path Merging

```
int f(int key, int data[7])
{
    int i, count = 0;
    for (i=0; i<7; i++) {
        if (key==data[i])
            count++;
    }
    return count;
}
```

- Exponential growth in expression size!

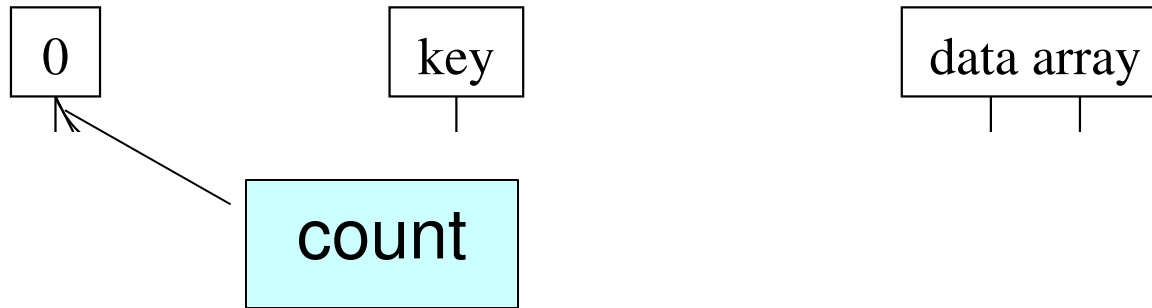
Combinational Equivalence: Key Ideas

- Symbolically simulate to compute functionality.
- **Use an efficient representation for the symbolic simulation, e.g., BDDs or circuit-like structure for SAT.**
- Find equivalent points to use as cutpoints to simplify the problem.

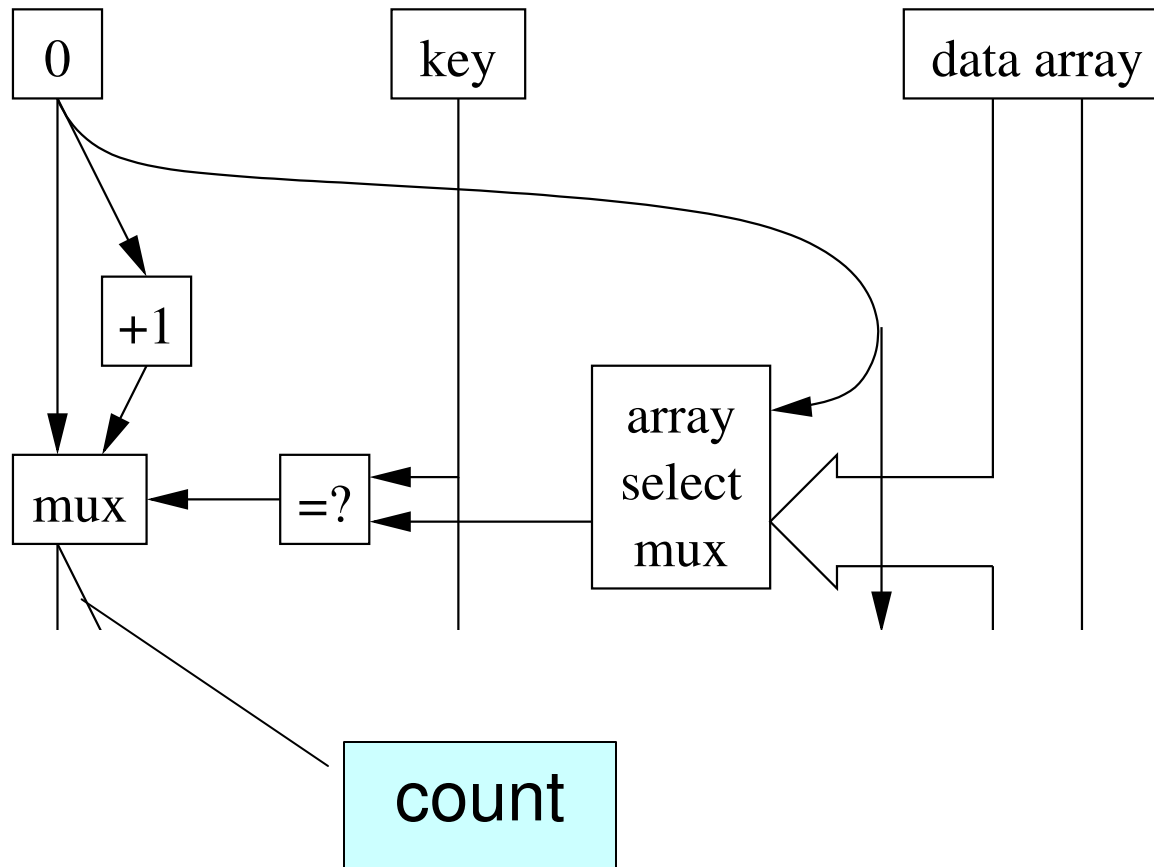
Shared Graph Representation

- Use a maximally shared combinational circuit graph as representation of functionality.

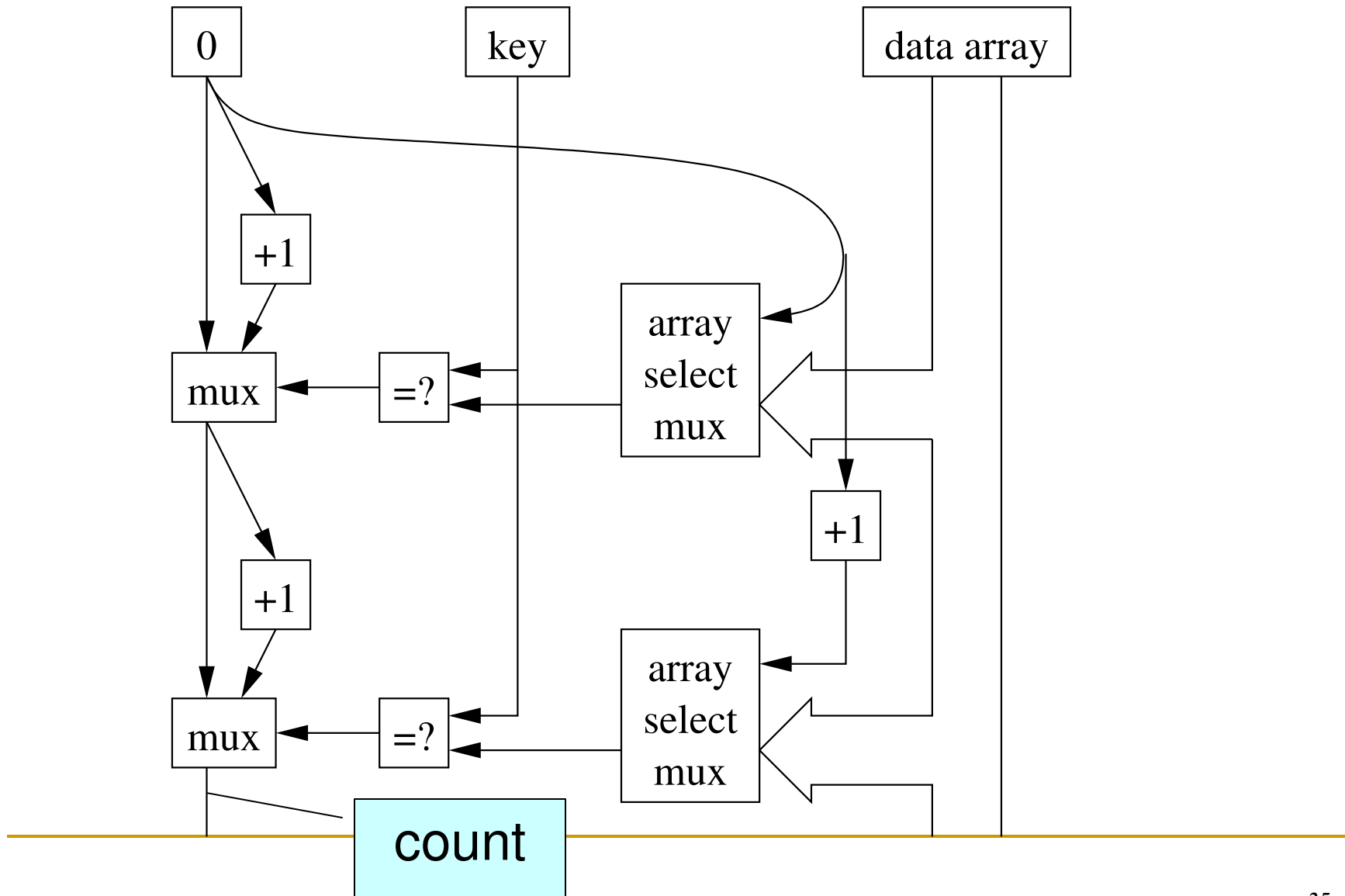
Shared Graph Representation



Shared Graph Representation



Shared Graph Representation



Shared Graph Representation

- Use a maximally shared combinational circuit graph as representation of functionality.
- Graph structure grows linearly (in size of unrolled program).
- Result is essentially a synthesized combinational circuit.

- Still has potential problems for very complex software

Combinational Equivalence: Key Ideas

- Symbolically simulate to compute functionality.
- Use an efficient representation for the symbolic simulation, e.g., BDDs or circuit-like structure for SAT.
- **Find equivalent points to use as cutpoints to simplify the problem.**

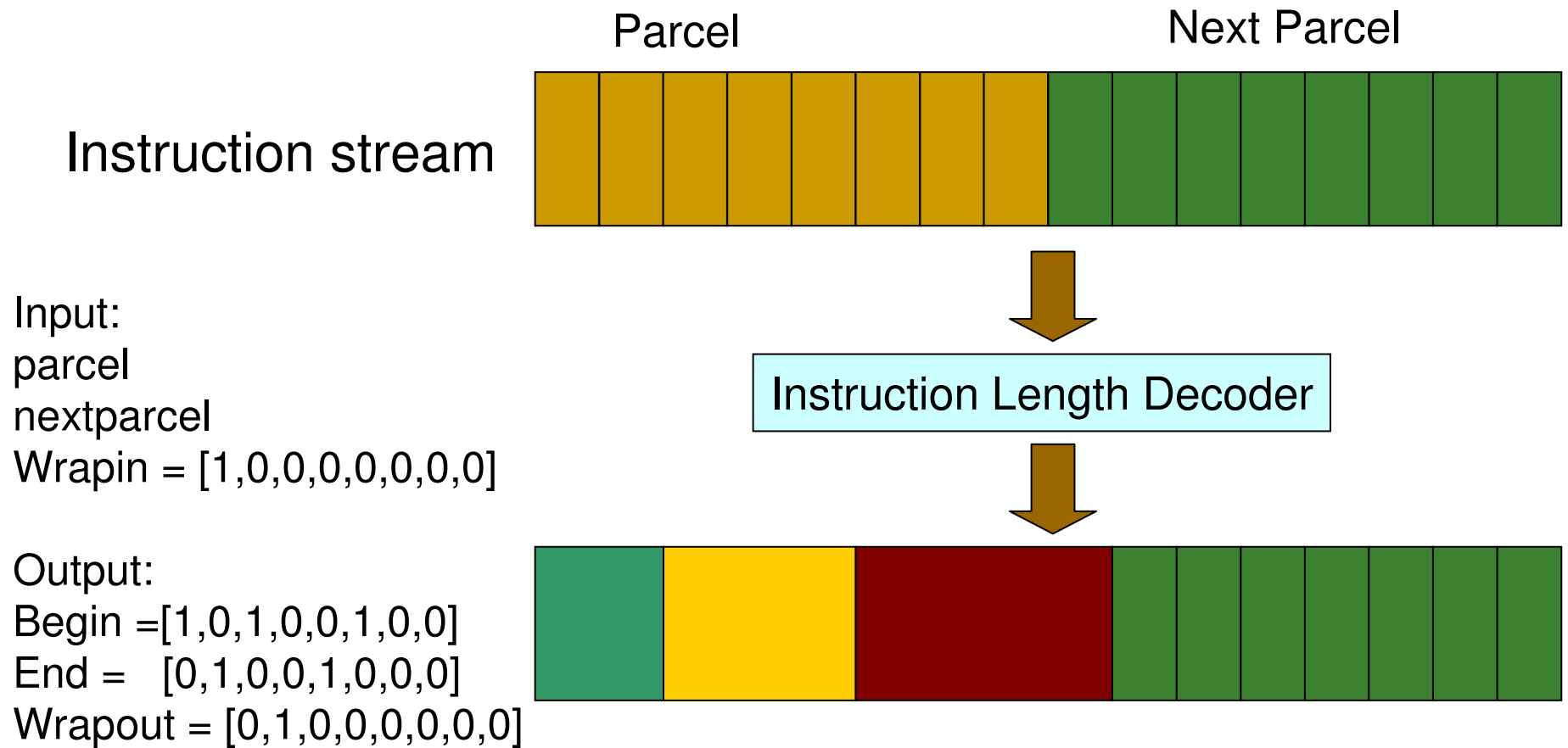
Early Cutpoint Insertion

- Find and insert cutpoint **during** symbolic simulation of software, not after synthesizing an equivalent circuit.
- Reduces blow-up, allows using BDDs to represent path conditions.
- Therefore, can handle much more complex branching and looping conditions.

Case Study: IA-32 Instruction Length Decoder

- Challenge problem suggested by Robert Jones of Intel Corporation.
- IA-32 has very complex instruction encoding:
 - Variable length instructions from 1 to 15+ bytes
 - Prefixes, over-rides of field lengths, etc.
- Instruction length decoder marks instruction boundaries in an instruction buffer, in a single cycle.

IA32 Instruction Length Decoder



Software Model of ILD

```
1. while (wrap < PARCEL_SIZE) {
2.     begin[wrap]=1; /* Start of instruction */
3.
4.     /* Set default sizes. */
5.     operand_mode = INIT_OPERAND_MODE;
6.     address_mode = INIT_ADDRESS_MODE;
7.
8.     get_next_byte();
9.     ret = handle_prefixes();
10.    /* If there were any prefixes, get the next byte for opcode. */
11.    if (ret) get_next_byte();
12.
13.    if (current_byte != ESCAPE) handle_one_byte_opcodes();
14.    else { /* Escape to two-byte opcode */
15.        get_next_byte(); /* Skip over the escape code. */
16.        handle_two_byte_opcodes();
17.    }
18. }
```

Hardware Model of ILD

- All decoding is in parallel
- A priority-encoding network to decide which blocks of ILD logic are the valid ones:
$$\text{valid}(P_m) \text{ iff } \text{valid}(P_n) \wedge m = n + \text{length_from}(n)$$
- Optimized by using script.rugged (SIS)

Verification Challenges

- Software
 - ❑ Easy to describe the functionality
 - ❑ Serial
 - ❑ Exponential number of paths
 - ❑ Very complex control flow
- Hardware
 - ❑ Complicated, RTL circuit
 - ❑ Highly parallel (one cycle)

Effect of Early Cutpoints

Example	Linear Building BDD		Early Cutpoints	
	Time(s)	Mem(MB)	Time(s)	Mem(MB)
EX20-8	0.28	61	0.11	58
EX20-16	89.01	1746	0.24	60
EX20-32		mem out	0.53	64
EX20-64		mem out	1.35	72
EX97-8	1.46	92	0.51	64
EX97-16	1187.72	1800	1.10	73
EX97-32		mem out	2.35	95
EX97-64		mem out	5.41	136
EX251-12	309.18	1843	0.64	66
EX251-16		mem out	1.09	71
EX251-32		mem out	7.45	170
EX251-64		mem out	16.81	327

Experimental Results

hw-CBMC vs. Early Cutpoints

Example	hw-CBMC		Early Cutpoints	
	Time(s)	Mem(MB)	Time(s)	Mem(MB)
TOY-8	6.84	38	0.01	56
TOY-16	502.59	522	0.02	56
TOY-32	time out		0.06	56

Future Directions

- Heuristics for finding cutpoints
- Program analysis and optimization techniques to expose parallelism
- Handling more complex control flow
- Handling dynamic memory
- False inequivalence handling
- Integrating with other techniques to remove cycle-accuracy assumption