Modeling Soft Error Effects Considering Process Variations

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Abstract

This paper addresses the aggregated effects of two types of variations that contribute to the reliability degradation. The first one is the increasing level of process variation; the second one is one particular type of environmental variation – the radiation-induced soft error. Their simultaneous presence can cause large negative performance impact. We present a statistical approach to model the generation and propagation of a transient soft error inside combinational circuits considering the existence of inter-die channel length variation in CMOS digital circuits. Experiment results have demonstrated that channel length variation can significantly aggravate the soft error effect, which can be accurately evaluated using the proposed methodology.

1. Introduction

VLSI circuits are becoming more susceptible to radiation-induced soft errors [1]. A soft-error, or a “single event upset (SEU)”, is caused by high-energy cosmic particles striking the sensitive region of semiconductor devices. Due to technology scaling, soft error rate (SER) in random logics increases rapidly. It is predicted that SER in logics will surpass that in unprotected memories [2] in 2011. Soft errors can be categorized into two types depending on the impact location. A strike on a sequential device may cause an accidental bit flip if the deposited charge exceeds a critical value $Q_{\text{crit}}$. A strike on a combinational gate may cause a temporary voltage swing at the gate output, called a “single event transient (SET)”, or a “transient”. It will become a stable error if captured by a flip-flop (FF). In this paper, we only address SET due to particle strikes at combinational nodes.

Meanwhile, since the control of critical device parameters is becoming more difficult, significant variations in key device parameters have resulted, causing large uncertainties in design performance. Process variations can be classified as inter-die and intra-die variations. Inter-die variation refers to the variation of the device parameters among different locations on the same die. The impacts of process variations on timing and power have been extensively researched [3][4]. Process variations may also aggravate soft error effect, which has not been thoroughly investigated.

This paper models the impact of inter-die variations in channel length on the soft error effects. We first consider the generation of a SET incurred by a particle strike on a combinational node. The channel length variation in the struck transistor leads to a distribution of the generated transient, modeled as a random variable. We further consider the propagation of the transient as a random variable transformation, when the channel length variations in the encountered gates further extend the distribution.

Not all transient errors at combinational nodes can become observable errors due to three masking effects: logic masking, latching window masking and electrical masking [2]. The largest impact of process variation is on the electrical masking effect. We model the variations in the strength of a transient due to a particle strike at a logic gate with distributional channel length, as well as the variations in the propagation behavior when a transient propagates through a logic gate whose channel length observe a similar distribution. Our discussion is limited to the following scope: (1) we consider only inter-die variations; (2) we consider only strikes by particles with fixed energy levels. Natural particle activity can be taken into consideration as a second random variable that is totally uncorrelated.

The rest of the paper is organized as follows: section 2 reviews the basic mechanisms of single event transients; section 3 models the impact of inter-die channel length variations on single event transient generation and propagation; section 4 presents simulation results; and section 5 concludes the paper.

2. Single-Event-Transient

When a cosmic particle enters a material, it deposits charges along its path. The linear energy transfer (LET) is used to relate the incident energy to the charge deposition. It is defined as the energy loss of the particle per unit path length. In bulk silicon, a...
typical charge collection depth \( \lambda_c \) is ~2\mu m for an LET of 1 MeV-cm²/mg, and an ionizing particle deposits \( q_d \) =10.8fC of charge along each micron of its track. Thus a particle with LET=1 deposits ~21.6fC of charge [5]. Natural particle activity is characterized by “LET spectrum” – the number of particles detected on a unit area per unit time as a function of LET. When a particle strikes the drain area of a CMOS transistor, the collected charge will form a transient double-exponential current flow in at the p-n junction[6]:

\[
I(t) = \frac{Q}{(\tau_a - \tau_p)}(e^{-t/\tau_a} - e^{-t/\tau_p})
\]

where \( Q = \text{LET}^{\alpha} \lambda_c q_d \), is the total deposited charge; \( \tau_a \) is the “collection time constant”; and \( \tau_p \) is the “ion track establishment time constant”. Typical values are 1.64x10⁻¹⁰ sec for \( \tau_a \) and 5x10⁻¹¹ sec for \( \tau_p \).

Figure 1 shows a particle striking the PMOS transistor in an inverter and the corresponding linear RC model. \( I(t) \) is the transient current; \( R_D \) is the equivalent resistor of the NMOS transistor network. When the input is 1, the PMOS transistor may be temporarily shorted, causing a transient voltage \( V(t) \) at the output. In [7], an approximate closed-form expression of \( V(t) \) was obtained:

\[
V(t) = \frac{Q}{C_n} e^{-t/\tau_a} \left[ \frac{1}{\tau_a - \tau_p} \right] \left( e^{-t/\tau_a} - e^{-t/\tau_p} \right) \left( \frac{1}{\tau_a - 1/\tau_a} \right) - 1
\]

where \( \tau_a = C_n R_D \), \( C_n \) is the sum of the output capacitance \( C_o \) and the load capacitance \( C_l \). The contribution of \( \tau_p \) is ignored as it is small compared to \( \tau_a \). Solving \( dV(t)/dt = 0 \), we can easily find that \( V(t) \) reaches its peak at time \( t_m \):

\[
t_m = \frac{\tau_a \tau_p}{\tau_a - \tau_p} \ln \left( \frac{\tau_a}{\tau_p} \right)
\]

and the peak value \( V_G \) is given by:

\[
V_G = V(t_m) = \frac{Q}{C_n} \left( \frac{\tau_a \tau_p}{\tau_a - \tau_p} + \frac{\tau_p}{\tau_a} \ln \left( \frac{\tau_a}{\tau_p} \right) \right)
\]

\[
= \frac{\lambda_c q_d (\tau_a \tau_p / (\tau_a - \tau_p) + \tau_p / \tau_a \ln (\tau_a / \tau_p))}{\tau_a - 1/\tau_a}
\]

\[
\times \text{LET}
\]

Figure 1. Linear RC Model of a Particle Strike

Hence \( V_G \) seems to be linear to the incident energy. However, our experiment shows that \( V_G \) can be better fit into a quadratic function of \( \text{LET} \). As shown in Figure 2, when a particle strikes the PMOS transistor in the inverter INV1 (a), \( V_G \) at its output is plotted against the incident LET (b). This can be explained as follows: when the particle energy is low, the dominant mechanism is the funneling effect, based on which (4) is derived; as the energy increases, the ion track shunting becomes significant [10], causing \( V_G \) to deviate from the straight line. As \( \text{LET} \) further increases to \( \text{LET}_{\text{MAX}} \), \( V_G \) saturates to the \( V_{dd} \). Considering that \( V_G = 0 \) at \( \text{LET} = 0 \), \( V_G \) can be expressed as:

\[
V_G(\text{LET}) = \begin{cases} k \cdot \text{LET} \cdot (\text{LET} + q) & \text{LET} < \text{LET}_{\text{MAX}} \\ V_{dd} & \text{LET} \geq \text{LET}_{\text{MAX}} \end{cases}
\]

3. SET under Channel Length Variation

In this section, we first use one example to show that channel length variation can have significant impact on SET, and then we present models of transient generation and propagation; and we revisit the example to demonstrate how to apply our model.

3.1. An Example: Inverter Chain

The circuit in Figure 3(a) consists of 6 identical inverters driving a flip-flop DFF0. The input A is a stable 1 when a particle with \( \text{LET}=10 \) strikes the PMOS transistor in INV0. If the channel length \( l \) of all inverters is of nominal value \( l_0=0.13 \mu m \), the transient at the struck node n1 has a magnitude \( V_t=893.7mV \). Simulation shows that it will be attenuated by the inverter chain and will not reach node Y.
Assuming the channel lengths observe a normal distribution ($\mu_0$ and $3\sigma$), we ran Monte Carlo simulation of 1000 samples and produced the histogram of the transient magnitude at $Y$ (Figure 3(c)): in a large number of samples, a transient is able to reach $Y$. If $DF0$ is sensitive to a 600mV input, we will observe a latched error in $DF0$ in 42% of all samples. Therefore, channel length variation can reduce the noise margin and worsen the soft error effect. Error analysis will be significantly inaccurate without considering process variations.

![Example circuit (Chain of 6 inverters)](image)

(a) Example circuit (Chain of 6 inverters)

![Generated and propagated transient at Y](image)

(b) Generated and propagated transient at $Y$

![Histogram of transient heights at node Y](image)

(c) Histogram of transient heights at node $Y$

Figure 3 An Inverter Chain Example

### 3.2. Modeling Transient Generation

When a particle strikes a combinational gate, the strength of the transient varies with the channel length $l$. Figure 4(a) shows the voltage at the output of $INV1$ when $I$ varies, from where we can see that $V_{G}$ increases with $I$. This is caused by the changes in the parameters $k$ and $q$. We observed that $q$ is relatively independent of $I$, hence $V_{G}$ can be expressed as:

$$V_G(l) = f_G(l) = k(l) \cdot \exp(-\frac{(l-l_0)}{\sigma l})$$

(8)

$$= (a_G \cdot l + b_G) \cdot \exp(-\frac{(l-l_0)}{\sigma l}) = K \cdot l + K_s$$

(8)

![Voltage Variation with Channel Length](image)

(a) Voltage Variation with Channel Length

![Linear Dependency of Transient Magnitude on Channel Length Variation](image)

(b) Linear Dependency of Transient Magnitude on Channel Length Variation

Figure 4 Variation of Transient Generation

We only keep the first-order term in the Taylor expansion because the variation is typically small. So $V_{G}$ is a linear function of $l$ for fixed LET value, which has been verified by simulations for several LET values in Figure 4(b). From the boundary condition that $V_G=0$ when $l=0$, $b_G$ in (8) should be zero.

The inter-die channel length variation is generally considered to be a normal distribution $N(\mu_0, \sigma_0^2)$, whose probability density function (PDF) is:

$$f_l(l) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(l-\mu)^2}{2\sigma^2}\right)$$

(9)

From the rules of functions of random variables[13], it can be easily proved that $V_G$ also has a normal distribution $N(\mu_G, \sigma_G^2)$, and:

$$\mu_G = \mu_0$$

(10)

In our experiment, if the distribution of channel length in $INV1$ is as shown in Figure 4(c), $V_G$ has a distribution as shown in Figure 4(d).

### 3.3. Modeling Transient Propagation

When $V_G$ propagates through the inverter $INV2$ (Figure 2(a)), if $INV2$ has nominal channel length, the distribution of $V_P$ can be studied in three regions:

1. $V_G<h_1$, $V_P=0$: the probability is given by the cumulative PDF of $V_G$: $P(V_P=0)=F_{V_G}(h_1)$;
2. $h_1 \leq V_G \leq h_2$, $V_P=\alpha V_G + \beta V_P^d$, as a linear function of $V_G$, has a normal distribution: $P(V_P)=N(\mu_P, \sigma_P^2)$, with mean $\mu_P=\alpha \mu_G + \beta$ and standard deviation $\sigma_P=\alpha \sigma_G$;
3. $V_G>h_2$, $V_P=V_P^d$: the probability is given by the probability of $V_G>h_2$, i.e. $P(V_P=V_P^d)=1-F_{V_G}(h_2)$.

Figure 5(a) shows two scenarios of the transient propagation. In both cases, $h_1=0.667V$, $h_2=0.962V$, and the nominal $V_{G(d)}$ are in the transition region ($h_1 \leq V_{G(d)} \leq h_2$), so the nominal $V_P$ is determined by the linear relation in (6). However, the variations in $INV1$ cause a certain portion of $V_G$ to reach beyond the transition region. In case (1), $V_G<h_1$, close to the upper limit of the transition region, for some values of $l$, $V_G(l)$ will exceed $h_2$, causing $V_P$ to be saturated to $V_P^d$. This results in a discontinuity in $V_P$ distribution and a spike at $V_P=V_P^d$. Similarly, in case (2), $V_G(l)=0.75V$, close to the lower limit of the transition region, the discontinuity in $V_P$ distribution is on the left side and the spike appears at $V_P=0$.

The above discussion is valid when $INV2$ has nominal channel length. In reality, the variation in $INV2$ affects its DC characteristic. As shown in Figure 5(b), both $h_1$ and $h_2$ increase with channel length $l$. For small variations in $l$, both $h_1$ and $h_2$ are approximately linear functions of $l$, as supported by the experiment results shown in Figure 5(c):

$$h_1(l) = a_1 \cdot l + b_1$$

(11)

$$h_2(l) = a_2 \cdot l + b_2$$

Hence, both $h_1$ and $h_2$ have normal distributions. This variation will cause further deviation in $V_P$ distribution. When considering inter-die variation, the channel lengths of $INV1$ and $INV2$ are 100% correlated. Finding the $V_P$ distribution is equivalent to
solving the following problem: “Given a random variable \( l \), whose PDF is defined in (9), and three linear functions \( V_G(l), h_1(l) \) and \( h_2(l) \), defined in (8) and (11), find the distribution of \( V_P \) as defined in (6), which is a function of random variables \( V_G, h_1 \) and \( h_2 \).”

First, we calculate the transient observed at node \( Y \) when node \( n_1 \) is struck by a particle with fixed LET value: \( L E T < L E T_{M A X} \). The transient at INV1 output \( V^{(1)}_P \) is given by (6). The transient at INV2 output \( V^{(2)}_P \) can be calculated as by applying (6) to \( V^{(1)}_P \):

\[
V^{(2)}_P = V^{(1)}_P - \frac{h_2 - h_1}{V_{dd}} (V^{(1)}_P - h_1) - \frac{h_2 - h_1}{V_{dd}} (1 + R) \frac{R}{R^+} \tag{17}
\]

where \( R = (h_2 - h_1)/V_{dd} \). Note that (17) holds only when \( V^{(2)}_P \) is in the transition region: \( h_1 \leq V^{(2)}_P \leq h_2 \), or \((1 + R) h_1 \leq V^{(2)}_P \leq h_1 + R h_2\). It is easily proved that \((1 + R) h_1 > h_1\), and \(h_1 + R h_2 < h_2\). So the transition region of the 2 inverters is narrower than a single inverter. The interval during which the transient propagates linearly shrinks. Repeating the computation, we obtain the transient magnitude at \( Y \) in the transition region:

\[
V^{(n)}_P = V^{(n-1)}_P - \frac{h_2 - h_1}{V_{dd}} (V^{(n-1)}_P - h_1) - \frac{h_2 - h_1}{V_{dd}} (1 + R) \frac{R}{R^+} \tag{18}
\]

where \( N \) is the logic depth from the struck node to the endpoint \( FF_0 \) \((N=5 \text{ in the example})\). The transition region on the DC characteristic of the inverter chain is:

\[
h_1 \frac{1 - R^x}{1 - R} < V_P < h_1 \frac{1 - R^x}{1 - R} + R^x \cdot V_{dd} \tag{19}
\]

The transition region of a well-designed inverter is usually very narrow, i.e. \( R \) is very small, so \( R^N \) decreases rapidly as \( N \) increases. In our example, \( R=0.27 \) when \( l=l_h \) so \( R^N \sim 1.4 \times 10^{-3} \) for \( N=5 \). As \( N \) becomes large, the transition region converges to a single point: \((V_G(l)=h_1/(1-R))\). This means that \( V_P \) is either 0 or \( V_{dd} \) depending on the magnitude of the generated transient \( V_G \) at the struck node:

\[
V_P = \begin{cases} 0, & V_P < h_1/(1-R) \\ V_{dd}, & V_P \geq h_1/(1-R) \end{cases} \tag{20}
\]

Since \( V_G \), \( h_1 \), and \( R = (h_2 - h_1)/V_{dd} \) are linear functions of \( l \), when \( n_1 \) is struck, the probability for the generated transient of not being able to reach \( Y \) is equal to the probability that: \( V_G(l) - h_1/(1-R) < 0 \). It can be rewritten as a quadratic function of \( l \):

\[
D(l) = (K - a_1) l^2 + (a_2 - K - b_1) l + b_3 > 0 \tag{21}
\]

where \( K = a_G \cdot L E T + q \), \( a_2 = a_2 - a_1 \), \( b_3 = 1 - b_2 = 1 - (b_2 - b_1) \); and \( a_G \), \( q \) are as defined in (8); \( a_1, a_2, b_1, b_2 \) are as defined in (11). We will show in section 4.1 that all the parameters are positive and \((a_1 \cdot b_1') > 0\). The discriminant of \( D(l) \) is a quadratic function of \( K \):

\[
\Delta(K) = b_1^2 \cdot K^2 - (2 \cdot b_2 \cdot a_1 + 4 \cdot a_1 \cdot b_1') \cdot K + a_1^2 \cdot b_1'^2 \tag{22}
\]

Now we will discuss several possible situations.

1) \( \Delta(K) < 0 \) (Figure 6(a)). \( D(l) > 0 \) for all values of \( l \) because the coefficient of \( l^2 \) \((K-a_1)\) is positive. The discriminant of the quadratic function \( \Delta(k) \) is given by:

\[
\Delta_x = 16 \cdot (a_1 \cdot b_1 \cdot a_1 \cdot b_2 \cdot a_1 \cdot b_1')^2 \tag{23}
\]

Obviously, \( \Delta_x \) is positive. Therefore, \( \Delta(K) \) always has two real zeros \( K_1, K_2 \) and the range of \( K \) for \( \Delta(K) < 0 \) is \( K_1 < K < K_2 \). In reality, we can prove that \( K > K_1 \) is
always true because \( K < K_f \) causes the two zeros of \( D(l) \) to be negative. Therefore, if \( K < K_f, A(K) < 0 \), and \( D(l) \) is always positive. This means that for any value of channel length, the transient at \( n_l \) will not be able to propagate to \( Y \). Knowing that \( K = a_G \cdot \text{LET} \cdot (\text{LET} + q) \), we conclude that if the particle’s energy is sufficiently small, the incurred transient will not reach \( Y \) regardless of the actual channel lengths of the inverters. Also, \( D(l) \) shifts toward the x-axis as \( K \) increases and will intersect with x-axis when \( K \) reaches certain value.

2) \( A(K) = 0 \). This determines minimum \( K_{th} \) for \( D(l) \) to have real zeros as well as \( \text{LET}_{th} \), the threshold of the particle’s LET for a transient to reach \( Y \).

3) \( A(K) > 0 \) (Figure 6(b)). \( D(l) \) has two zeros \( l_1, l_2 \):

\[
l_1, l_2 = \frac{(K - b_l - a_l) \pm \sqrt{(K - b_l - a_l)^2 - 4 \cdot K \cdot a_l}}{2 \cdot K}
\]

(24)

**Figure 6 Error Probability vs. \( \text{LET} \) and \( l \)**

It is obvious that \( D(l) > 0 \) when \( l < l_1 \) or \( l > l_2 \). In other words, a strike at \( n_l \) can reach \( Y \) only when \( l < l_1 < l_2 \). Our experiment has shown that \( l_2 \) is typically much greater than the upper bound of channel length. Therefore, \( P_e \), the probability of an error at \( Y \), is equal to the probability of \( l > l_1 \), or \( P_e = 1 - F_l(l_1) \), where \( F_l(l) \) is the CDF of the channel length distribution. As the incident LET increases, \( K \) increases and \( D(l) \) shifts downwards, which causes the root \( l_1 \) to decrease. Consequently, the error probability \( P_e = 1 - F_l(l_1) \) increases with LET because \( F_l(l) \) is a monotonically increasing function of \( l_1 \). This means a particle with higher energy causes higher error probability.

Using the conclusion derived above, the distribution in Figure 3(c) can be explained. In the example, \( l_1 > l_0 \), so at nominal channel length \( l_0 \), \( V_G \) at \( n_l \) is not enough to propagate to \( Y \). With the variation in channel length, \( V_G \) is able to reach \( Y \) in the samples where \( l \) surpasses \( l_1 \); in samples where \( l \) remains below \( l_1 \), \( Y \) remains undisturbed. There are a small number of samples in which \( V_G \) is less than \( V_{dd} \). This is not consistent with the prediction that the transient at node \( Y \) is either \( 0 \) or \( V_{dd} \). It is because the transition region is still of a finite width in the experiment.

4. Experimental Results

In this section, we present some experimental results. All simulations are performed on a 0.13\( \mu \)m inverter cell, designed using the Predictive Technology Model (PTM) for bulk CMOS [11][12].

4.1. SET Generation and Propagation

First, we characterize the inverter and determine all parameters in the model (Table 1). The parameters are dependent on many factors, including the load capacitances, supply voltage, as well as the operating conditions. The values listed in Table 1 are for the inverter driving a load of 50fF in normal operation conditions, and the \( V_{dd} \) is set to 1.2V. We now briefly describe how to obtain these parameters.

**Table 1 Parameter Characterization**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_G )</td>
<td>0.0293</td>
<td>( \text{V} \cdot \mu \text{m} \cdot (\text{MeV} \cdot \text{cm}^2/\text{mg})^{-1} )</td>
</tr>
<tr>
<td>( b_G )</td>
<td>0.0001</td>
<td>( \text{V} \cdot (\text{MeV} \cdot \text{cm}^2/\text{mg})^{-1} )</td>
</tr>
<tr>
<td>( q )</td>
<td>11.92</td>
<td>( \text{MeV} \cdot \text{cm}^2/\text{mg}^{-1} )</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>1.368</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0.489</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>2.732</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.607</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>1.364</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>0.118</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( a_3' )</td>
<td>0.882</td>
<td>( \mu \text{m} )</td>
</tr>
</tbody>
</table>

To determine the values of \( a_G \) and \( b_G \) in equation (8), we first fix \( l \) and disturb the transistor with particles with LET ranging from 0 to \( \text{LET}_{MAX} \), the coefficients \( k \) and \( q \) in equation (5) are obtained by fitting the transient magnitude \( V_G \) to a quadratic function for the specific value of \( l \). Figure 2(b) shows \( V_G \) can be well fit into the quadratic function of \( \text{LET} \). Then we vary \( l \) from 100nm to 160nm and find different values of \( k \). The parameter \( a_G \) and \( b_G \) is then determined by linear fitting. Due to boundary condition that \( V_G = 0 \) at \( l = 0 \), \( b_G \) is negligibly small (\( 10^{-6} \)).

To determine \( a_1 \), \( b_1 \), \( a_2 \) and \( b_2 \) in (11), we set up the experiment circuit in Figure 2(a). We first fix the \( l \) and attack the transistor in INV1 with particles with LET from 0 to \( \text{LET}_{MAX} \), the coefficients \( h_1 \) and \( h_2 \) are measured from the DC characteristics of INV2. Then we vary \( l \) from 100nm to 160nm and find different values of \( h_1 \) and \( h_2 \) as functions of \( l \). The parameters \( a_1 \), \( b_1 \), \( a_2 \) and \( b_2 \) are determined by linear fitting.

Next, we verify that the distribution of \( V_G \) is normal and find its mean and standard deviation. We fix the incident LET at 10 MeV-cm\(^2\)/mg and run Monte Carlo simulation of 1000 samples with normally distributed channel length (\( \mu_l = 0.13 \mu \text{m} \), \( \sigma_l = 0.013 \mu \text{m} \), as shown in Figure 4(c). The histogram and fitted normal distribution of \( V_G \) are plotted in Figure 4(d), and the mean and deviation are shown in the first two rows in Table 2, where the “Calculation” data is obtained from equation (10), only 5.1\% (for \( \mu \)) and 6.9% (for \( \sigma \)) different from the data measured from simulation.

We also verify the distribution of the propagated transient through INV2 when its channel length is fixed.
at $l_0$. The result is shown in the lower part of Table 2. In both cases in Figure 5(a), we compared the mean and deviation in the transition region calculated using our model to the results measured from simulation. We also compare the probability of $V_F=V_{dd}$ in case (1) and $V_F=0$ in case (2). Although the relative errors of our transient propagation model remain reasonably low (6.8%~10.4%), they are higher than the error of the transient generation modeling, because the modeling error in $V_G$ distribution accumulates to $V_F$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean ($\mu_c$) (mV)</th>
<th>Calculation</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SET</td>
<td>887.5mV</td>
<td>900.5mV</td>
<td>1.3%</td>
</tr>
<tr>
<td>PROP</td>
<td>887.5mV</td>
<td>950.5mV</td>
<td>5.5%</td>
</tr>
</tbody>
</table>

Table 2 Distribution of $V_G$ and $V_F$

4.2. Case Study: Inverter Chain

Using the data in Table 1, the three cases in 3.4 can be detailed as:

1) $A(K)<0$ if $K_1<K<K_2$, where $K_1=4.392$ and $K_2=6.14$. Note that $K_1<K$ always holds to guarantee at least one of the two zeros $l_{1,2}$ of D(l) is positive. Solving $K_1=K_2=9.695$ for $K=K_2$, we obtain one positive real zero $LET_{th}=9.695$ (the other root $LET_{th}$ is negative). So if the particle’s $LET$ is less than $LET_{th}$, the transient at n1 will not reach Y, regardless of channel length.

2) $A(K)<0$ if $K=K_{th}=6.14$. This determines the threshold LET that might cause an error: $LET_{th}=9.695$.

3) $A(K)<0$ if $K>K_{th}$: for any $LET>LET_{th}$, D(l) has two real zeros $l_{1,2}$. And the transient will reach node Y when $l_1<l<l_2$. In other words, for any value of $l$, there exists a minimum value $LET_{th}$ such that when $LET>LET_{th}$, the transient will reach Y.

Figure 7 Experiment Results of the Inverter Chain

We did two experiments on this example circuit. First, we validate the value of $LET_{th}$ for different $l$. The results are shown in Figure 7(a). The “Simulation” data is obtained by gradually increasing the LET value for a fixed $l$ until an error is observed in DFF0 during the simulation. The relative error between simulation and calculation is on average 5.6%.

Second, we compare the error rate measured in DFF0 with our model. The error rate is plotted against $LET$ in Figure 7(b). The “Calculation” data is obtained by first determining the value of $l$ for a fixed $LET$, then calculating the error rate as $1-F(l)$.

The “Simulation” data is obtained from SPICE Monte Carlo simulation: for each LET value, we use 10,000 samples of varying channel length and count the number of errors observed in DFF0. It can be seen that our model’s prediction matches the experimental result very well. Specifically, the simulation data showed that the error rate remains zero until $LET=9.8$ MeV·cm$^2$/mg, whereas the predicted threshold value is $LET_{th}=9.695$ MeV·cm$^2$/mg.

5. Discussions and Conclusions

In this paper, we developed statistical models of the generation and propagation of radiation-induced transient with the presence of inter-die channel length variation. Further work is needed to complete the model by considering intra-die process variations; and how to conveniently apply the model to large circuit systems remains a big challenge.

References